Every Student Counts

Elementary
Professional Development Guide
Year 2 - Day 3

Geometry and Measurement

Iowa Department of Education

Elementary Session – Facilitator's Plan
Year 2 Day 3

Content Goals:

NCTM- Geometry Standard

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

K-2

- Describe, name, and interpret relative positions in space and apply ideas about relative position;
- Describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
- Find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.

3-5

- Describe location and movement using common language and geometric vocabulary;
- Make and use coordinate systems to specify locations and to describe paths;
- Find the distance between points along horizontal and vertical lines of a coordinate system.

NCTM- Measurement Standard 3-5

Understand measurable attributes of objects and the units, systems and processes of measurement

3-5

- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.

Apply appropriate techniques, tools, and formulas to determine measurements

3-5

- Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;
- Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;
- Select and use benchmarks to estimate measurements;
- Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;
- Develop strategies to determine the surface areas and volumes of rectangular solids.

Principle Focus: Teaching

Process Goal: Reasoning and Proof Standard K-12

Overall Teaching Goal:

1. Teaching and learning mathematics through problem solving

Activity	Description for facilitator	Time (min)	Teacher Masters (TM-) & Materials
1) Welcome	Discussion of agenda, goals and sessions for the day Highlight the process standard and principle: teaching Graph data on #1	10	**TM- 1** Daily Plan **TM- 2** Yearly Outline Content standard diagram to look at measurement
2) Readings	Readings 1-2 Discussion: Random groupings; Create a graphic representation of the teaching standards for math and summarize using reflection questions after teams share.	45	Poster sheets **TM-3** Reflection Questions for Day 3
3) Linear Measurement PBIT K-2	Create a list of key vocabulary words for geometry and measurement Discussion of measurement progressions and research on measurement Levels of development and assessment of measurement From Small to Tall *Problem Solving and Reasoning* Grade 2 Scavenger Hunt page 34 in Measurement 3-5	60	**TM- 4** Measurement Activity Sequence **TM-17** MDP Longer than, Shorter than Review of linear measurement concepts *Reasoning and Problem Solving Grade 2* page 19 Unit Lengths page 42 Body Building page 43
4) Area Explorations K-5	Read article: Developing Spatial Sense through Area Area without Numbers Activity 1 Rearranging Area Activity 2 Measuring Length Activity 1	75	**TM- 5** MDP preview **TM – 13** MDP Article from Measurement 3-5 Bridges Units "Measurement " and "Area Explorations" Tape measures Meter/yard sticks
	• LUNCH	45	
5) PBIT K-5	Exploring area on a geoboard	20	Geoboards, rubber bands ,geodot paper, paper squares **TM – 12** PBIT Geoboards

Activity	Description for facilitator	Time (min)	Teacher Masters (TM-) & Materials
6) Building Boxes 3-5	Exploring volume related to surface area	35	Blocks Centimeter Grid paper Scissors Building Boxes from *Measurement 3-5* page 77 **TM-16** PBIT-Volume and surface area Centimeter cubes Scotch Tape
7) Discourse that Promotes Conceptual Understanding	Participants will be given tools to assist in strengthening questioning skills during the various phases of a problem-based lesson.	60	Question cards. Video of teachers during lesson questions focus **TM-6** Phases of Lesson **TM – 7** Role Sheet **TM 14** Discourse That Promotes Conceptual Understanding **TM-15** PBIT for 2nd grade lesson
8) Phases of a Lesson	Participants will be given a set of tools for use with teachers that will enhance understanding of problem-based tasks.	30	**TM 6** Phases of a Investigative Lesson **TM 7** Examining Teaching and Learning in a Mathematics Classroom Question Cue Cards Video of a lesson from one of these sources: "Mathematical Thinking" from Math Trailblazers training video
9) Processing readings	Reading summary and discussion on question 3 and the role of vocabulary	15	**TM- 8** Reflection Questions from

Activity	Description for facilitator	Time (min)	Teacher Masters (TM-) & Materials
	words in learning concepts. Revisit list made and discuss insights.		Reading Assignment for Day 3 **TM-9** Problem Solving Standard
10) Reflection /homework assignments	Evaluation and reading assignments	15	**TM-10** Reading Assignment for Day 4 **TM-11** Practice Assignment for Day 4

Facilitator's Tool for Planning the Session

Equipment and materials the **facilitator** should bring**:** The instructor will need overhead geoboard.

Equipment and materials **participants** should bring: Geoboards, scissors, and scotch tape.
Handouts
Bridges Units "Measurements"
     Handouts 1,2,3
Bridges Units "Area Explorations"
     Handout 1
TM- 1A and 1B Daily Plan
TM- 2 Yearly Overview
TM- 3 Reflection Questions for Day 3
TM- 4 Measurement Activity Sequence Diagram
TM-5 MDP Preview
TM- 6 Phases of a PBIT lesson
TM-7 Role Sheet
TM-8 Use TM- 3
TM-9 Problem Solving Standard
TM-10 Reading and Practice Assignments for Day 4
TM-12 Geoboard PBIT
TM-13 MDP Review
TM-14 Features of Discourse That Promote Conceptual Understanding
TM-15 PBIT Investigating Area
TM 16 PBIT Volume and Surface Area
TM 17 MDP Measurement
Question Cues

Overheads
TM- 1 Daily Plan
TM- 3 Reflection Questions for Day 3
TM- 4 Measurement Activity Sequence Diagram
TM-5 MDP Preview
TM-9 Problem Solving Standard
TM-13 MDP Review

Materials
Scissors
Centimeter Graph paper
Tape
Geoboards
Yard sticks
Meter sticks
Tape
Poster sheets and markers

Activity 1: Welcome and Overview
Time: 5 minutes

Overview and Rationale
Participants will record their number one choice on the reflection question using a post it note graph on a poster on the wall. Each person puts his post it note on the part of the Teaching Standards for Mathematics that they think is the beginning point for working with teachers. This will be part of the morning activities and get everyone thinking about the day and the Teaching Principle.

Conducting the Activity

1) Use an overhead of the Daily Plan to share the content, process, principle and assessment foci for the day.
2) Refer to the yearly outline for more detail.
3) Discuss of agenda, goals and sessions for the day.
4) Recruit 8 teachers for the role play in the afternoon and give them copies of "Discourse for Conceptual Understanding" to review during lunch in preparation. Assign parts and practice with them during lunch.
5) Highlight the process standard and principle: teaching
6) Examine the data on the participant graph on the question from homework.

Materials
Overhead of **TM-1** and **TM-2**
8 copies of "Discourse for Conceptual Understanding" by Elham Kazemi from *Teaching Children Mathematics*, March 1998:410-414.

Graph Question
Question: If you were to change just 10% of how you teach math now, what aspect recommended in the standards for teaching mathematics will you focus on? If you are not teaching now, what would you recommend teachers focus on first? Why?

Choices:

Problem-Based Instructional Tasks
Teacher's Discourse
Student Discourse
Learning Environment
Tools for Learning
Analyzing Teaching and Learning

TM 1A Daily Plan

Time	Activity
9:00 am	Welcome
9:10	Readings Discussion
9:55	Break
10:05	Linear Measurement
10:55	Area Explorations
12:10 PM	LUNCH
12:55	Exploring Geometry on a Geoboard
1:15	Building Boxes
1:50	Break
2:00	Questioning and Discourse
3:00	Reading and practice discussions
3:15	Evaluation and new homework assignments
3:30	dismiss

TM 1B: Daily Overview Diagram

Every Student Counts means . . .

**Teach for Understanding
and
Focus on Meaning**

**Problem-Based Instructional Tasks &
Teaching through Problem Solving**

Meaningful Distributed Practice of Concepts, Skills, and Problem Solving

Today's Goals . . .

Content Goal: Measurement Standard K-5
Process Goal: Problem Solving
Principle Focus: Teaching

Today's Objectives . . .
- *Understand and apply principles of teaching measurement.*
  - *Investigate and evaluate classroom discourse and the teacher's role in effective questioning.*

TM-2 :Elementary Year Two Outline 2005-06

	Day 1 September 13-14, 2005	**Day 2** November 1-2, 2005	**Day 3** January 24-25, 2006	**Day 4** March 28-29, 2006
NCTM Content Standard 1	**Algebra** Representing using math symbols; Modeling; Analyzing change	**Geometry** Analyzing 2D and 3D shapes and characteristics	**Geometry** Coordinate geometry	**Geometry** Transformations and visualizing
NCTM Content Standard 2		**Measurement** Understand measurable attributes of objects and the units, systems, and processes of measurement	**Measurement** Apply appropriate techniques, tools, and formulas to determine measurements	
Mathematical Activities	**K-2 Algebra** • Illustrate general principles and properties of operations, such as commutativity, using specific numbers; • Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations. • Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.	**Measurement K-2** • Recognize the attributes of length, volume, weight, area, and time; • Compare and order objects according to these attributes; • Understand how to measure using nonstandard and standard units; • Select an appropriate unit and tool for the attribute being measured.	**Measurement 3-5** • Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes; • Select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles; • Select and use benchmarks to estimate measurements; Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms; Develop	**Geometry K-2** • Recognize and apply slides, flips, and turns; • Recognize and create shapes that have symmetry. • Create mental images of geometric shapes using spatial memory and spatial visualization; • Recognize and represent shapes from different perspectives; • Relate ideas in geometry to ideas in number and measurement; • Recognize geometric shapes and structures in the

	Day 1 September 13-14, 2005	Day 2 November 1-2, 2005	Day 3 January 24-25, 2006	Day 4 March 28-29, 2006
	• Describe qualitative change, such as a student's growing taller; • Describe quantitative change, such as a student's growing two inches in one year **3-5** • Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers; • Represent the idea of a variable as an unknown quantity using a letter or a symbol; • Express mathematical relationships using equations. • Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions. • Investigate how a	**Geometry** **K-2** • Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes; • Describe attributes and parts of two- and three-dimensional shapes; • Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes. Geometry 3-5 • Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; • Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids; • Investigate, describe, and reason about the results of subdividing, combining, and	strategies to determine the surface areas and volumes of rectangular solids **Geometry** **K-2** • Describe, name, and interpret relative positions in space and apply ideas about relative position; • Describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance; • Find and name locations with simple relationships such as "near to" and in coordinate systems such as maps. Geometry 3-5 • Describe location and movement using common language and geometric vocabulary; • Make and use coordinate systems to specify locations and to describe paths; • Find the distance between points along horizontal and vertical lines of a coordinate	environment and specify their location. **Geometry** **3-5** • Predict and describe the results of sliding, flipping, and turning two-dimensional shapes; • Describe a motion or a series of motions that will show that two shapes are congruent; • Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs. Build and draw geometric objects; • Create and describe mental images of objects, patterns, and paths; • Identify and build a three-dimensional object from two-dimensional representations of that object; • Identify and draw a two-dimensional representation of a three-dimensional object;

	Day 1 September 13-14, 2005	Day 2 November 1-2, 2005	Day 3 January 24-25, 2006	Day 4 March 28-29, 2006
	change in one variable relates to a change in a second variable; • Identify and describe situations with constant or varying rates of change and compare them.	transforming shapes; • Explore congruence and similarity; • Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.	system.	• Use geometric models to solve problems in other areas of mathematics, such as number and measurement; • Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.
NCTM Principle	Equity	Technology	Teaching	Learning
NCTM Process Standard	Connections Reasoning and Proof	Reasoning and Proof Representation	Problem Solving Reasoning and Proof	Communication Representation
Assessment	Formative use of Meaningful Distributed practice	Interviews for conservation of measurement concepts	Questioning strategies for Problem-based Instruction	Journaling Analyzing student work
Technology /Manipulative tools		Java Applets on the Internet; nets	Geoboard explorations	Link cubes and isometric drawings

Activity 2 – Discussion of Readings #1 and #2
Time: 45 minutes

Overview and Rationale:
Bring focus to the day by having teams and tables refine, explain and justify their choices for the graph.

Conducting the Activity:
1. Each person at the table should share perspective on the choice on the graph. Instruct participants to create a graphic that will incorporate the ideas shared and include the relationship between the parts of the standards for teaching mathematics. Instruct the audience:

"Create a graphic representation for the teaching standards for math as team. Reflect the relative importance of each standard to each team member in your drawing. Share selected reflections and post team drawings for the day."

2. Have teams share and explain choices. Emphasize the importance of each of the standards and the interdependence of them.

3. Audience results from the training were overwhelmingly favoring Analyzing Teaching and Learning as the most popular focus for beginning with teachers.

Materials:
Posters paper
Markers
Tape to post
TM 3 Reflections on Reading Assignments for Day 3

TM – 3 Reflections on Reading Assignments for Day 3

1. If you were to change just 10% of how you teach math now, what aspect recommended in the standards for teaching mathematics will you focus on? If you are not teaching now, what would you recommend teachers focus on first? Why?

2. What key characteristics of effective mathematics teaching in your opinion were found in the readings? Which ones are the most difficult to master?

3. Why do you think students have difficulty with measurement concepts? What kind of experiences would help students with them?

4. Vocabulary words are an important part of geometry. How can we enhance student learning of math vocabulary?

Activity 3
Times 60 minutes

Overview and Rationale: Participants will participant in linear measurement activities.

Conducting the Activity:
1. Create a list if key vocabulary words for geometry and measurement. Ask participants to think about what grade level the students should have understanding of each word. Save the list for later discussion.
2. Discuss measurement activity sequence. Give examples of each phase and how it would look in a classroom. Emphasize that in each measurement component (length, area, volume) the children need to have all of the experiences for understanding.
3. Using the MDP Shorter than, Longer than, model the comparison sequence.
4. Following the lesson from *Navigations Problem Solving and Reasoning Grade 2* have participants break into groups of 4-6 to make a baby, a ten-year-old, or a adult basketball player. Model the questions with the group that are listed in the lesson. Discuss how these questions help children move through the measurement sequence**.**

Materials:
Navigations Reasoning and Problem Solving Grade 2 From Small to Tall page 19
Unit length page 42
Body Building Page 42
Large paper
Tape
Markers
TM-4 Measurement Activity sequence
TM- 17 MDP Longer than, Shorter than

TM-4
Measurement Activity Sequence

Comparisons: Direct → Indirect measures

↓

Use of units: Nonstandard → Standard units

↓

Use of instruments: Nonstandard → Standard units

Student-made → Conventional

Distributed Practice and Questions: Longer than, Shorter than

Grade Level: K- 1

Big Idea(s): Students will recognize and compare attributes of length.

MDP Activity 1	MDP Activity 2	MDP Activity 3	MDP Activity 4	MDP Activity 5
Show the children an object such as a pencil. **"Can you find something in our classroom that is longer than this pencil?"**	Show the same object as Day 1. "Can you find objects in our classroom that is shorter than this object?"	Recall lists made from the previous days of items that are shorter than and longer than the object (pencil).	Show the children an object that is longer than the object on day 1 such as a yard stick. **"Can you find something in our classroom that is longer than this yard stick?"**	Have both the pencil and the yard sick available. "Can you find an object that is longer than the pencil, but shorter than the yard stick?
Questions: **How do you know it is longer?** **Is there any other way we could determine that this object is longer?** **Demonstrate the ideas given.**	Questions: How do you know that it is shorter? Is there any other way we could determine that the object is shorter?	Questions: **Choose on object from the longer than list and one object from the shorter than list. Ask which item is longer? Why?** **Repeat with different object.**	Questions: **How do you know it is longer?** **Is there any other way we could determine that this object is longer?** **Demonstrate the ideas given.**	Questions: **How do know the object is both longer than the pencil and shorter than the yard stick?** **Can you compare the items any other way?**
Assessment	Assessment	Assessment	Assessment	Assessment

TM 5: MEANINGFUL DISTRIBUTED PRACTICE Preview

Distributed Practice, Questions, and Assessment: Grade Level/Class 5

Big Idea(s) Area is the measure of units that cover a 2D space. Choose the unit to measure with that gives the needed accuracy.

Day One	Day Two	Day Three
Practice Activity 1 Show two irregular concave polygons. Ask Which is larger? How could we tell? Use overhead triangles to cover both shapes and compare the number.	Practice Activity 2 Ask students to consider the area of the surface of two books or other objects similar in size. Is one larger than the other? What would be an appropriate unit to measure the area of the surface? Give these choices: Index card Triangle pattern block Square post it notes	Practice Activity 3 Ask students to consider the area of the classroom. How could we compare the size of their classroom to another one? What might we use to cover the floor? Show a model of a square foot made of construction paper. Have students measure one length and discuss questions below.
Questions: What would happen if I used blue rhombi to cover the shapes? Can you predict how many that would be (two triangles=one rhombus)?	Questions: Measure with all of them. Which is better to use? Why? What accuracy is needed to determine differences in size? What is the advantage to having a square shape when measuring a rectangular figure?	Questions: Would we have to cover the floor or can you think of a way to estimate the number of square feet we would need another way?

TM 13: MEANINGFUL DISTRIBUTED PRACTICE Review

Distributed Practice, Questions, and Assessment: Grade Level/Class 5

Big Idea(s) Area is the measure of units that cover a 2D space. A formula can help us find area of polygons.

Day One	Day Two	Day Three
Practice Activity 1 Using a geoboard, show two rectangular polygons with an area of 12 each. Ask students, do these have the same area? How do you know?	Practice Activity 2 Using a geoboard, show two triangles with an area of 6. Ask students, do these have the same area? How do you know?	Practice Activity 3 Using a geoboard, show an irregular shape that can be subdivided into squares, triangles and rectangles to find area.
Questions: What is the greatest perimeter possible with an area of 12?	Questions: Can we find the area of all triangles using this method?	Questions: What is the area of this shape? How can we find it? Model the solutions suggested.

Activity 4– Area Explorations
Time: 75 minutes

Overview and Rationale:
Participants define and discuss uses of area. They will be asked to order the area from smallest to largest and discuss various ways to determine area of shapes.

Conducting the Activity:
1. Ask participants to read "Developing Spatial Sense Through Area". Ask participants to note the five foundations of measurement skills and look for them in the following activities.
2. Follow the manual for *Bridges To Classroom Mathematics* in the unit "Area Without Numbers" for activities 1-2. Discuss how participants can find the area with tools and without tools.
3. If time , participants may want to also do Rearranging Area Activity 2 and Measuring Length Activity 1 from the Bridges manual.

Materials:
Bridges Manual tape measure
Meter/yard sticks

Activity 5-Exploring Area on a Geoboard
Time 20 minutes

Overview and Rationale: Participants experience a Problem-based instructional task to explore area using a geoboard.

Conducting the Activity:
1. Check for understanding by having participants make various squares on a geoboard. Discuss how many different size squares that can be made on a geoboard. Show the types of squares that can and cannot be used to measure area in this PBIT.
2. Ask participants to work through the PBIT as described. Emphasis the algebra component that can be connected to this lesson.
3. Discuss the reflection questions at the end of the lesson. Having student samples is a good way to stimulate discussion.

Materials:
Overhead projector
Overhead geoboard/rubberbands
Geoboards
Geodot paper
Square paper
Crayons
TM-12 Problem based instructional task: Inside, Outside

TM-12 PROBLEM-BASED INSTRUCTIONAL TASK : Inside , Outside
LESSON PLAN

_____

Objective/Benchmark: Students will develop an understanding of area as the number of squares units needed to cover a region.
Title: Inside, Outside
Grade Level/Course: 1-2
Pre-Requisite Knowledge: Students can identify and make squares on a geoboard
NCTM Standard(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

Materials Needed:

Audio-visual: Overhead
Manipulatives: Overhead geoboard, geoboards, rubberbands, geodot paper, square paper ,crayons
Literature:
Technology/Software:
Other:

Adapted from " Inside, Outside" in The Super Source ETA/Cuisenaire

Main Lesson Development:
Launch:
Ask children how to use one rubber band on the geoboard to make the largest possible square possible square. Ask them to think of the rubber band as the edge of the geoboard. Using a different colored rubber band, ask children to make a design similar to yours (Make a simple closed square shape).
Tell the children you have made two regions. One region is inside the smaller shape. The other region is the space between the smaller shape and the big rubber band square.
Ask children which region they think covers the more space.
Discuss ways you could find how much space each region covers. Show the students how to use geoboard squares. Have children count along as you fill the shape with square paper mark. Write Inside-4. Count the squares outside the shape the same way, and record. Write Outside-12. Discuss about where there is more space, inside or outside the shape.
Explore:
Have students made the largest square on their geoboard. Use a rubber band of a different color to make a smaller shape inside the big square. The smaller shape must be made up of the small geoboard squares. Ask them to think about which region has more space: the inside or the outside. Make a drawing on geodot paper of how your geoboard looks. Cover the inside of

your smaller shape with the square paper markers .Count how many you used. Record on your paper. Cover the rest of the squares on your geoboard with square paper markers of another color. Record how many were on the outside region.

Post the children's drawings in three groups: those that had more squares on the inside, those that had more squares on the outside and those that had the same number of squares on the inside and the outside.

Begin a class chart that looks like this:

How many inside?	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
How many outside?															

Summarize:

Ask the children if anyone had a drawing with one square inside. Ask how many they had for the outside. Begin filling in the chart with students data.

Ask the following questions as you fill in the chart:

- After you found out the number of squares on the inside of your shape, did you know the number of squares outside you shape? Explain
- What do you notice about the chart?
- What do you see when you look the posted shapes?
- What do you notice about shapes that can be covered by the same number of squares?
- Was it easier to find the area of some shapes than others?
- Was there a point before we finished our chart that you knew what some of the missing numbers would be? Explain

Modifications/Extensions:

- Have children compare how several different shapes can all have the same area. Challenge them to explore to find all the ways.
- Ask children to predict missing numbers in the chart and analyze for algebraic patterns.

Checking for Understanding (Formative Assessment)

- **What will you assess?** Children will understand the relationship of area when the inside of the area changes either larger or smaller.
- **How will you assess it?** Ask children to write/draw what happens to the outside shape when a shape gets bigger. Explain their answers.

------------------ (REFLECTION AFTER TEACHING THE LESSON) ------------

- **How did the students perform?**
- **How will you use this information to guide future instructional decisions?**

Activity 6 – Exploring Volume Grades 3-5

Overview and Rationale:
Participants will experience a problem-based task that is an example of using the measurement principles of direct measurement using nonstandard units.

Conducting the Activity:
This lesson is from *Navigating Through Measurement Grades 3-5* page 77-82. Follow the instructions in the book or the following lesson to conduct the activity with adults. Compile results as a group on the overhead for each box. Discuss predictions for next box before creating the box. Be sure to note the various counting strategies students and adults use to find the total. Tie the counting strategy to the development of the formula for volume.

Materials:
TM 16 Problem-Based Instructional Task Volume and Surface Area
Roll of tape
Pair of scissors
Five sheets of centimeter grid paper
Overhead of student chart included in *Navigating through Measurement* for this activity

TM 16 Problem-Based Instructional Task Lesson Plan

Lesson Topic: Volume and surface area
Grade Level/Course: 5
Objective: Students will develop strategies for calculating the number of cubes needed to fill a box, understanding volume and analyzing change in volume related to surface area.
Pre-requisite Knowledge: Students should be acquainted with rectangular prisms.
NCTM Standard(s): (shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

Materials Needed:

> **Audio-visual:**
> **Manipulatives:** 2 centimeter cubes
> **Literature:**
> **Technology/Software**:
> **Other**: Roll of tape, pair of scissors, five sheets of centimeter grid paper

This lesson is from *Navigating Through Measurement Grades 3-5* page 77-82.
Launch:
   Say to children: Imagine you work for Box-M-Up Packaging Company. In this company, boxes are created without lids first. The lids are added later. In today's class, you will make scale models of boxes for your company.
We are going to use grid paper to design the boxes.

Using one of the sheets, model how to create a box from a 9 x 11 rectangle. Have students create 5 sheets with these dimensions. Using one sheet, show how to cut one square from each corner. Make first box by taping corners. Fill with cubes and find total number it will hold.

After sufficient time, ask students to share strategies for finding the total number. Did they count every cube? Did the use the number in the top row and the number of rows?

Explore:
With students in pairs, have teams design a new box by cutting a larger 4 x 4 square from each corner. Ask students to predict whether the number of cubes needed, will be more or less than the last box. Record predictions on the board. Ask students to make the next box and see if their predictions were accurate. Continue recording information on the chart paper for the rest of the rectangle sizes.

Summarize:
Ask students to describe the patterns on the chart. Patterns should include:
- The length decreases by 2 as each corner is cut.
- T width also decreases by 2 as each corner is cut.
- The height increases by 1 each time.
- The number of cubes is the length times the width times the number of layers or height.

Define *volume* as the attribute they were measuring. Ask for methods of finding the volume that they used. When they have agreed the number of cubes is length x width x height, write the formula on the board: *volume=length x width x height or volume= (area of the base) x height* in a rectangular solid. Pose the question if this method would work for any solid. What about a sphere? A triangular prism? A cube?

Modifications

Use chart from Building Boxes A that focuses on volume and does not record surface area.

Extensions

Have students make boxes from a square beginning with a 9 x 9 grid. Ask them to compare results with the first exploration.

Checking for Understanding (Formative Assessment)

- **What will you assess?**
  - Students show understanding of finding volume of rectangular prisms.

- **How will you assess it?**
  - Students will record in a journal answers to these questions:
    - How could you find the number of cubes that would fill a box without actually filling it?
    - What would be the dimensions of a box that you could build from a 10 x 10 rectangle by cutting one square from the corner? What would be its volume?

------------------ (REFLECTION AFTER TEACHING THE LESSON) ------------

- **How did the students perform?**
- **How will you use this information to guide future instructional decisions?**

Activity 7 – Discourse That Promotes Conceptual Understanding
Time: 60 minutes

Overview and Rationale:
Teaching in a problem-based classroom requires close attention to the discourse that occurs in the classroom. Participants will be given tools to assist in strengthening questioning skills during the various phases of a problem-based lesson.
.
Conducting the Activity:
1. Review the factors that impacted high student achievement in classrooms as indicated by the research summarized in the article and on TM 14.
2. Tell the audience the other features of a student-centered classroom were present in both classrooms. The major differences were found on these four points.
3. Noting the first point, have the first role play occur. This is the classroom with "high press" which is asking students to explain and demonstrate their understanding. Following the role play, point out the teacher's persistence in asking students' to show answers by shading, write mathematical statements and explain the connections to the drawing and equations.
4. Have second group perform the role play of the teacher who does not have high press for learning. Have participants discuss in small groups what the teacher could have done in this situation. After a few minutes, ask for a few suggestions from the group.
5. Refer to the second point on TM 14 and ask for the third group to show a teacher that uses a different method to respond to errors. Following the role play, discuss any questions the audience may have about the process she used. Note the focus on the groups determining the answers and not on the individual student. The teacher uses the error to involve the whole class in further discussion.
6. Bring up the fourth group and have them continue the story of the teacher with low press. Note how this teacher put words into the student's mouths and explained for them. Pose the question: What questions/ procedure would you recommend with this group of boys? Let table groups discuss for a few minutes and then have a large group sharing.
7. Summarize by noting the last two features of discourse that promote understanding and asking for examples of them from the audience.

Materials:
TM 14 Features of Discourse that Promotes Understanding
Overhead with 9 squares for the students to demonstrate their thinking
Overhead markers

TM- 14 Features of Discourse That Promote Conceptual Understanding

- **Explanations consist of mathematical arguments**

- **Errors offered opportunities to reconceptualize a problem and explore contradictions and alternative strategies**

- **Mathematical thinking involved understanding relationships among multiple strategies**

- **Collaborative work involved individual accountability and reaching consensus through mathematical argument**

Kazemi and Stipek, 1997. "Pressing Students to be Thoughtful"

Activity 8- Phases of Lessons
Time: 15 minutes

Overview and Rationale
Participants will be given a set of tools for use with teachers that will enhance understanding of problem-based tasks.

Conducting the Activity:
1. Share the tools included here are useful when helping teachers know the behaviors that occur in each part of the Problem-Based Instructional Task Lesson. Teachers are often not clear on what kind of information can be shared in the launch or how to summarize.
2. Model how these tools can be used to promote better understanding. Have audience choose to view a lesson from the lens of a student, a teacher or the task using TM 7. Watch the video and stop after each phase to discuss what occurred using the questions from TM 7.
3. Hand out the Question Cue cards to be made into bookmarks for teachers to use as a guide for the phases of the lesson.

Materials:
TM 6 Phases of a Investigative Lesson
TM 7 Examining Teaching and Learning in a Mathematics Classroom
Question Cue Cards
Video of a lesson from one of these sources:

"Mathematical Thinking" from Math Trailblazers training video

TM- 6: **Phases of an Investigative or Problem-based Task Lesson**

Launch

Get students mentally ready to work on the task.

Suggest a simpler version of the task.

Brainstorm strategies.

Estimate possible solutions.

Ask questions about the task for clarity and understanding.

Explore

Listen carefully to students as they work.

Provide hints or ask questions to guide thinking.

Observe and assess understanding.

Encourage testing ideas they may have.

Suggest extensions for those that finish early.

Instruct students to find a second method to confirm answer.

Note student strategies you wish to highlight during discussion.

Summarize

Accept student solutions without evaluation.

Engage in class discussion that allows students to justify and evaluate results and lessons.

Identify rules, hypotheses, or future problems.

Engage all learners in the discussion.

Use praise cautiously and for process rather than right answers.

Summarize understanding and new knowledge explicitly.

Assessment

Ask students to reflect on learning with a similar problem.

Have students write how the problem is best solved.

Have students generate their own problem by changing one or more variables and solve it using a new strategy.

TM – 7: Examining Teaching and Learning in a Mathematics Classroom

Questions for Teacher Role Lens:

1. What roles does the teacher assume when problem-based instruction is used?

2. What does the teacher look like?

3. What does the teacher sound like?

4. How are any errors handled?

5. How do these roles support effective learning?

Questions for Student Role Lens:

1. What roles does the student assume when problem-solving?

2. What does the student look like?

3. What do the student explanations sound like? Are they more mathematical argument or simply procedural summaries?

4. What kind of discourse was evident in groups? Were children supporting their answers with mathematical arguments?

5. Did students make connections between strategies?

6. How do these roles support effective learning?

Questions for Task Lens:

1. What characteristics can be attributed to problem-based tasks that make them different from traditional problem solving?

2. What, if anything, makes this task engaging to the students?

3. What mathematics concepts and skills might a student employ when engaged in this type of task?

LAUNCH QUESTIONS

(Can students understand, define, formulate, or explain the problem or task? Can they cope with poorly defined problems?)

1. **What is this problem about?**
2. **How would you interpret that?**
3. **Would you please explain in your own words?**
4. **What do you know about this part?**
5. **Do you need to define or set limits for the problem?**
6. **Is there something that can be eliminated or that is missing?**

EXPLORE QUESTIONS

(Do students have an organized approach to the problem or task? How do they record? Do they use tools appropriately?)

1. **Where could you find the needed information?**
2. **What have you tried? What steps did you take?**
3. **How did you organize the information? Do you have a record?**
4. **Do you have a system? A strategy? A design?**
5. **Have you tried (tables, trees, list, diagrams, etc.)?**
6. **How would it look if you used these materials?**
7. **How would you research that?**
8. **Let's see if we can break it down. What would the parts be?**
9. **What if you used smaller numbers? Drew a diagram?**

SUMMARIZE QUESTIONS

(Do students see relationships and recognize the central idea? Do they relate the problem to similar problems previously done?)

1. **What is the relationship of this to that?**
2. **What is the same? What is different?**
3. **Is there a pattern?**
4. **What if you moved this part? Changed this to a different number?**
5. **Can you think of a counterexample?**
6. **Does anyone have the same answer but a different way to explain it?**
7. **Where else would this strategy be useful?**
8. **What were the mathematical ideas in this problem?**
9. **Can you convince us that that makes sense?**
10. **How did your model, diagram, representation help you solve this problem?**

CHECK FOR UNDERSTANDING

(Do students reach a result? Are there indications students learned some mathematics? Can they describe or depict strategies they are using?)

1. **What were the mathematical ideas in this problem?**
2. **What was one (or more) thing you learned from this problem?**
3. **What are the variables in this problem? The constants?**
4. **Is this the only possible answer? How do you know?**
5. **How would you check the steps you have taken or your answer?**
6. **How did you know you were finished?**
7. **How would you explain this process to a younger child?**
8. **Is there a general rule? Explain.**
9. **What strategies did you develop to solve this problem? Would they work for other problems?**
10. **What new questions does this problem raise for you?**
11. **Change one condition in the problem and solve the new problem. How did it affect the outcome?**
12. **Evaluate two solution methods and explain which one you prefer.**

Prepared by Sandi Campie

TM- 15 PBIT Investigating Area

Objective/Benchmark: Students will measure area using multiple copies of nonstandard units.
Title: Investigating Area
Grade Level/Course: 2
Pre-Requisite Knowledge: Students know the characteristics of rectangles.
NCTM Standard(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

Materials Needed:

Audio-visual: toy grazing animal
Manipulatives: square tiles, pattern blocks, overhead pattern blocks
Literature: Murphy, Stuart. ***Bigger, Better, Best!.*** Harper Collins, 2002.
Technology/Software:
Other: Masters with irregular shapes

Adapted from "Investigating Area" in <u>Growing With Mathematics Grade 2. Topic 2 lesson 7.6</u>

Launch:
Show a picture of two rectangular fields and pose the problem:

These two rectangles are fields of grass. A (name toy) wants to live in the bigger field because there would be more grass to eat. Which field should he choose?

Discuss how we could find out. Elicit ideas such as cover with pattern blocks, tiles, measure with a ruler, etc.

Explore:
Allow time for students to try these ideas out on own copies of the diagram. After sufficient time, let students discuss and share strategies using overhead copy of diagram and overhead pattern blocks. Provide each pair of students a new diagram with areas and ask them to determine the greatest and least area of the three rectangles.

Summarize:

Discuss these questions:

Which shape was best to use to cover the area? Why?

Which field is bigger?

What is area?

Modifications

Same as extension but use only two shapes cut from grid at a time and use tiles to cover them as verification.

Extensions:

Give a pair of students blank grid paper on card stock and instructions to create 6 shapes that have the areas 9, 10,11,12,13 and 14. Ask them to exchange with another pair of students and with the cards facedown put the new set in order of least area to greatest. Turn cards over to count squares on the grid to verify the correct order.

Checking for Understanding (Formative Assessment)

   What will you assess?

   Understanding of area and how to measure it.

   How will you assess it?

 Read story "Bigger, Better, Best". Ask students to suggest ways to figure out whether their classroom is bigger than the classroom next door. Write and draw about it in their math journal.

------------------ **(REFLECTION AFTER TEACHING THE LESSON)** ------------

   ▪ **How did the students perform?**

   ▪ **How will you use this information to guide future instructional decisions?**

Activity 9- Processing Readings and Homework
Time: 15 minutes

Overview and Rationale
Participants will process the readings and homework in a cooperative setting.

.
Conducting the Activity:
1. Participants will refer to the math vocabulary list generated earlier to discuss the importance of vocabulary in geometry. Ask participants to thinks about the difference between a definition of a word vs. understanding of that word. Linking this concept to reading vocabulary can be a powerful way for teachers to understand that children who use and communicate math vocabulary have a deeper understanding and enhance student learning.
2. Refer to the problem solving standard. Discuss how the understanding and use of vocabulary influences problem solving.

Materials:
Vocabulary Word list
TM -8 Reflection question #4 from TM 3
TM -9 Problem Solving Standard

TM-9
Problem Solving Standard
All students in PreK-12 should…

- Build new mathematical knowledge through problem solving

- Solve problems that arise in mathematics and in other contexts

- Apply and adapt a variety of appropriate strategies to solve problems

- Monitor and reflect on the process of mathematical problem solving

Activity 10
Time: 15 minutes

Overview and Rationale:
Close the session with evaluations and assignments.

Conducting the Activity:
Go over reading assignment and practice assignment for the next session. Ask participants to continue to think about the relationship between the Professional Development Model and Every Student Counts.

Materials:
TM-10 Reading and Practice Assignments

TM-10
Reading and Practice Assignments due Elementary Day 4

1. The principle focus for Day 4 will be the **Learning Principle**. Cognitive research tells us that students learn best when they have an opportunity to build on prior experience and knowledge, develop conceptual understanding during relevant problem solving experiences and use metacognitive reflection. **Read** the section in the Learning Principle for Grades K-12 in *Principles and Standards for School Mathematics*, pages 20-21.

   - **Reflection Question:** How can the Learning Principle enhance student engagement?

2. **Read** "Never Say Anything a Kid Can Say" by Steven C. Reinhart.

   - Choose a couple of strategies from pages 62-63 and explain the reasoning behind your choice.

3. **Read** Chapter 8 "Reflecting on Teaching Mathematics through Problem Solving" by Frances Curcio and Alice Artzt in *Teaching Mathematics through Problem Solving Grades PreK-6*, pages 127-147.

   - **Reflection Question:** How can teachers' knowledge, beliefs and goals and their problem solving behaviors influence students?

4. The Measurement Standard was the content standard focus for Day 3. **Read** "Measurement Standard" for Grades Pre-K – 12 (pp. 44-47) in *Principles and Standards for School Mathematics*.

   - **Reflection Question:** Why is measurement so important to the mathematics curriculum?

5. **Practice:** Refer to the article "Developing Spatial Sense through Area Measurement" by Elizabeth Nitabach and Richard Lehrer. Do the following:

   - Choose one of the Action Research ideas on page 186-187 that would be appropriate for your grade span. Prepare a short summary of your experience.
   - If you are teaching, reflect on the classroom discourse during a lesson. Set personal goals for yourself to improve your questioning. If you are not teaching, observe a lesson and watch for evidence of "press for learning". Summarize what you observed.